5

$$\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s}$$

6

$$\mathcal{L}^{-1}\left\{\int_s^\infty F(s)\,ds \text{ convergente.}\right\} = \frac{f(t)}{t}$$

7

$$\mathcal{L}\left\{f(t-\tau)\right\} = e^{-s\tau}F(s)$$

8

$$\mathcal{L}\left\{e^{at}f(t)\right\} = F(s-a)$$

$$\mathcal{L}\{t^3\} = \frac{3!}{s^4} \qquad e^{at} \quad te^{at} \quad t^2e^{at}$$

$$\mathcal{L}\{e^{at}t^3\} = \frac{3!}{(s-a)^4}$$

$$\mathcal{L}\{\cos(bt)\} = \frac{s}{(s^2+b^2)}$$

$$\mathcal{L}\{e^{at}\cos(bt)\} = \frac{(s-a)}{((s-a)^2+b^2)}$$

$$\mathcal{L}\{\operatorname{sen}(bt)\} = \frac{b}{(s^2+b^2)}$$

$$\mathcal{L}\{e^{at}\operatorname{sen}(bt)\} = \frac{b}{((s-a)^2+b^2)}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s+2}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s^2+2s)+2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+2s+1)+2-1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2+1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{(s+1)-1}{(s+1)^2+1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{(s+1)}{(s+1)^2+1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s+2}\right\} = e^{-t}\cos(t) - e^{-t}\operatorname{sen}(t)$$

1. $$\mathcal{L}\{af(t)+bg(t)\} = a\,\mathcal{L}\{f(t)\} + b\,\mathcal{L}\{g(t)\}$$

$$\mathcal{L}^{-1}\{F(s)\cdot G(s)\} = \int_0^t f(\tau)\,g(t-\tau)\,d\tau = f * g$$

convolución

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\cdot\frac{1}{s^2+1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} = \cos(t) \qquad \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} = \operatorname{sen}(t)$$

$$= \cos(t) * \operatorname{sen}(t) = \int_0^t \cos(\tau)\cdot\operatorname{sen}(t-\tau)\,d\tau$$

$$\begin{aligned}
&= \left[\int \cos(\tau)\Big(\operatorname{sen}(t)\cdot\cos(\tau) - \operatorname{sen}(\tau)\cos(t)\Big)d\tau\right]_0^t \\
&= \left[\operatorname{sen}(t)\int \cos(\tau)^2\,d\tau - \cos(t)\int \cos(\tau)\operatorname{sen}(\tau)\,d\tau\right]_0^t \\
&= \left[\operatorname{sen}(t)\int \tfrac{1}{2}\big(1+\cos(2\tau)\big)d\tau - \cos(t)\int \tfrac{1}{2}\operatorname{sen}(2\tau)\,d\tau\right]_0^t \\
&= \left[\frac{\operatorname{sen}(t)}{2}\int d\tau + \frac{\operatorname{sen}(t)}{2}\int \cos(2\tau)\,d\tau - \frac{\cos(t)}{2}\int \operatorname{sen}(2\tau)\,d\tau\right]_0^t \\
&= \frac{\operatorname{sen}(t)}{2}(t-0) + \frac{\operatorname{sen}(t)}{4}\big[\operatorname{sen}(2t)-0\big] + \frac{\cos(t)}{4}\big[\cos(2t)-1\big] \\
&= \frac{t\operatorname{sen}(t)}{2} + \frac{\operatorname{sen}(t)}{4}\big(2\operatorname{sen}(t)\cos(t)\big) + \frac{\cos(t)}{4}\big(\cos^2(t)-\operatorname{sen}^2(t)-1\big) \\
&= \frac{t\operatorname{sen}(t)}{2} + \frac{\operatorname{sen}^2(t)}{2}\cos(t) - \frac{\operatorname{sen}^2(t)\cos(t)}{4} + \frac{\cos^3(t)}{4} - \frac{\cos(t)}{4} \\
&= \frac{t\operatorname{sen}(t)}{2} + \frac{\operatorname{sen}^2(t)\cos(t)}{4} + \frac{\cos^2(t)\cos(t)}{4} - \frac{\cos(t)}{4} \\
&= \frac{t\operatorname{sen}(t)}{2} + \frac{\cos(t)}{4}\big(\underbrace{\operatorname{sen}^2(t)+\cos^2(t)}_{1}\big) - \frac{\cos(t)}{4}
\end{aligned}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} = \frac{t\operatorname{sen}(t)}{2} + 0$$

Teorema de Existencia y Unicidad de la Transformada de Laplace.

la función $f(t)$ de $\mathcal{L}\{f(t)\}$ debe ser de clase "A"

Una función es de clase "A" cuando

a) es de orden exponencial

b) seccionalmente continua

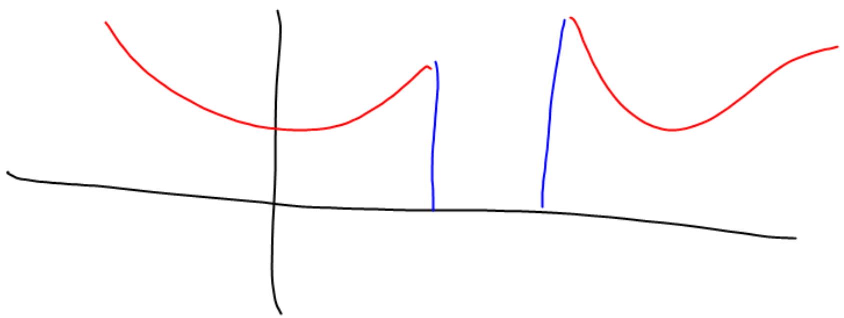
$$|f(t)| \leq Me^{At} \quad M, A \in \mathbb{R}$$

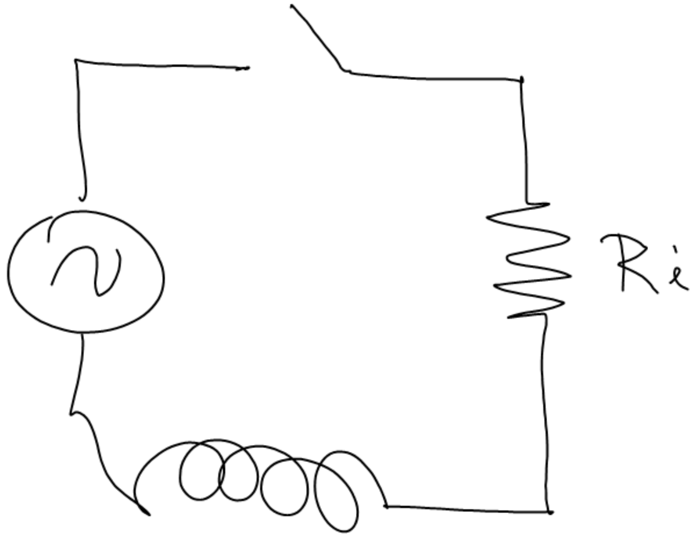
e^{x^2}

$$\Rightarrow e^{x^2} \quad |e^{x^2}| \leq Me^{AX}$$

b) Seccionalmente continua

- Aquella función que puede tener un número finito de discontinuidades en un intervalo cerrado $A \leq t \leq B$.





$$L\frac{di}{dt}$$

$$L\frac{di}{dt} + Ri = u(t-a)160\cos(60\pi t).$$

$$L\{e^{(t-5)}\} = \frac{e^{-5}}{s-1}$$

$$L\{e^{t}\} = \frac{1}{s-1}$$